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On the energy transfer in an optical coupler

I.-IOVITZ POPESCU[a], R. E. NISTOR[b*], E. PETRESCU[b]
[a]*National Institute for Lasers, Plasmas and Radiation Physics, PO Box MG7, 76900, Magurele, Romania*
[b]*Physics Department, Politehnica University, Bucharest, Romania*

The analogy between quantum mechanics and optics, as described in the foregoing paper [1], is used for the analysis of the total reflexion phenomenon. As an application, the photon propagation parameters in optical fibers is investigated. Also the energy transfer in an optical coupler is analysed.

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1. Introduction

In the present paper we will take the advantage of using the powerfull mathematical tool of the quantum mechanics to explain and to analyze quantitatively, in a most simple and rapid way, some otherwise complicated optical phenomena [1]. Specifically, by dealing with the problem of guided propagation of the light through optical circuits [2], and particularly in optical fibers, we will exclusively appeal to the photon concept only, avoiding any use of Maxwell equations for this purpose. The quantitative study has been conducted for a phosphate glass optical fiber due to its excellent properties as host material for optical amplifiers [3]

2. Total reflection on a dielectric interface

If it is desired that the photon should not cross the separation surface between two dielectric media (having the refraction index n_1 and n_2 respectively), it is necessary that (see Fig.1 [1]) p_{x2} to be imaginary (corresponding to the evanescent wave). In this case, from Eq. (19) of our previous paper [1], we get

$$p_{x2} = p_2\sqrt{1-\sin^2\theta_2} = p_2\sqrt{1-f\sin^2\theta_1} \qquad (1)$$

where

$$f = \left(\frac{n_1}{n_2}\right)^2 \qquad (2)$$

For the sake of easy computation, in the following analysis we shall consider a rectangular phosphate glass fiber, Fig.1, but the main conclusions remain valid also for the case of a circular cross section. In practice we meet two situations: a) the refraction indices n_1 and n_2 are both complex quantities (the case of metals) so that f is a complex quantity too; b) the refraction indices n_1 and n_2 are both real quantities, hence f is real. In the last case (b) from Eq. (1) we obtain an imaginary momentum p_{x2} if

$$1 - f\sin^2\theta_1 \le 0 \qquad (3)$$

that is if

$$\sin^2\theta_2 = f\sin^2\theta_1 = w \ge 1 \qquad (4)$$

where the incidence angle θ_1 is real, so that $\sin^2\theta_1 \le 1$. We now see that the last two conditions Eq. (3) and Eq. (4) can be fulfilled only if

$$\sin^2\theta_1 \ge \left(\frac{n_2}{n_1}\right)^2 \text{ and } f \ge 1 \text{ that is } n_2 \le n_1 \qquad (5)$$

Notice that in the case (a), when the refraction indices n_1 and n_2 are complex, the two conditions (5) remain still valid, this implying that the imaginary parts of n_1 and n_2 should be as low as possible. We also specify that, in the case of total reflection, the wave propagation at the interface between the two media is equivalent to the propagation through a thin metallic layer (see reference [5] in [1]).

3. Glass fiber caracteristics

In this section the major propagation characteristics of electromagnetic waves through an optical fiber will be derived exclusively on the basis of the photon concept. Let us consider a step-index fiber of rectangular cross section [4]. Thus, as it is well known, for any given wave guide mode, the transverse wave number is given by the corresponding critical wave number, namely

$$k_{x1} = k_c \qquad (6)$$

For instance, for a rectangular wave guide, as shown in Fig.1a, working in the fundamental TE_{10} mode, we have $k_c = \pi / 2a$. The total reflection of the signal occurs at the interface $x = \pm a$ (Fig. 1.b) of the core with the cladding [5].

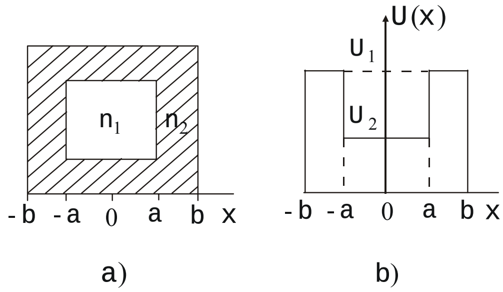

Fig. 1. a) Rectangular glass fiber; b) The glass fiber transversal potential.

Taking into account the relationships between momentum and wave number, $p = \hbar k$, photon energy and momentum, $E = pc$, and the expression $p_{x1,2} = p_0 n_{1,2} \cos\theta_{1,2}$, see [1], we obtain

$$k_c = \frac{E_{\min}}{\hbar c} n_1 \sqrt{1 - \sin^2 \theta_{1\min}} \tag{7}$$

where $\sin^2 \theta_{1\min} = \left(\frac{n_2}{n_1}\right)^2$. For energy values $E \geq E_{\min}$ and $\sin^2 \theta_1 > \left(\frac{n_2}{n_1}\right)^2$, we can write the relation between the transverse wave number k_{x1} and the photon incidence angle, θ_1, upon the lateral wall of the considered light guide, namely

$$k_{x1} = \frac{E}{\hbar c} n_1 \sqrt{1 - \sin^2 \theta_1} \tag{8}$$

On the other hand, using Eq.1, for the transverse wave number within the fiber cladding we have

$$k_{x2} = \frac{E}{\hbar c} \sqrt{n_2^2 - n_1^2 \sin^2 \theta_1} = i\chi \tag{9}$$

Let us consider the photon propagating through the optical fiber and meeting the transverse potential illustrated in Fig.1b. The corresponding wave functions built up in the fiber transverse section can be either even or odd, as presented in the following table.

Odd functions	Even functions
$\Psi_1 = Ae^{\chi x}$	$\Psi_1 = Ae^{\chi x}$
$\Psi_2 = B\sin k_{x1}x$	$\Psi_2 = B\cos k_{x1}x$
$\Psi_3 = Ce^{-\chi x}$	$\Psi_3 = Ce^{-\chi x}$

These represent proper functions of the Helmholtz operator, as shown in Eq.8 of [1]. The resulting continuity conditions for $x = -a$ lead to the following relationships

Odd functions	Even functions
$Ae^{-\chi a} = B\sin(-k_{x1}a)$	$Ae^{-\chi a} = B\cos(-k_{x1}a)$
$\chi Ae^{-\chi a} = k_{x1}B\cos(-k_{x1}a)$	$\chi Ae^{-\chi a} = -k_{x1}B\sin(-k_{x1}a)$

which, by division, become the following two transcendent equations [6]

Odd functions	Even functions
$\chi = -k_{x1}\cot(k_{x1}a)p$	$\chi = k_{x1}\tan(k_{x1}a)$

The resulting transcendent equations depend, via k_{x1}, on the photon incidence angle on the lateral wall, θ_1 [5]. Once determined this angle, we have also the corresponding transverse wave number in the core and in the cladding of the optical fiber. In continuation below will be discussed the numerical results obtained for an optical fiber having the parameters $2a = 250$ μm, $n_1 = 4$ and $n_2 = 3.5$ [7], for the frequency range 180-450 GHz and considering the cladding thickness $(b - a)$ high enough as to neglect its penetration.

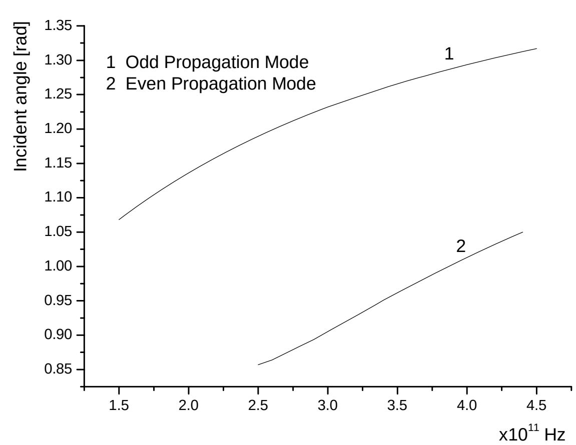

Fig. 2. The incidence angle θ_1 in terms of the frequency for $2a = 250$ μm, $n_1 = 4$ and $n_2 = 3.5$.

Fig. 2represents the incidence angle θ_1 in terms of the frequency as computed with the help of Eq. 7. Further, the penetration depth

$$\delta = \frac{1}{\chi} \tag{10}$$

presented in Fig. 3. has been computed with the help of Eq. 9 and the pair of the values energy – incidence angle from Fig. 2. The penetration depth can be increased by the presence of defects within the cladding [8]. Finally, the group velocity given in Fig. 5 in terms of frequency has been computed from the expression

$$v_g = \left(\frac{\delta p_{z1}}{\delta E} \right)^{-1} \tag{11}$$

where p_{z1} is

$$p_{z1} = \sqrt{p_0^2 n_1^2 - p_{x1}^2} \tag{12}$$

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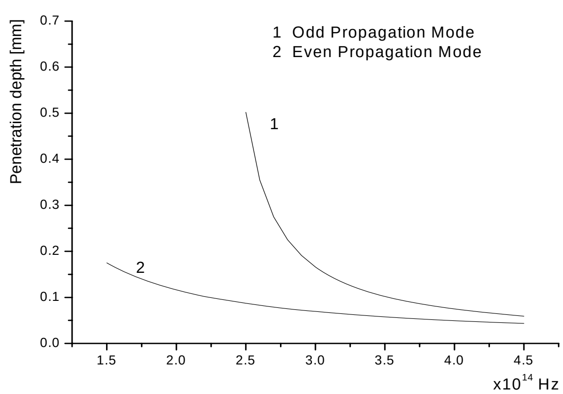

Fig. 3. Penetration depth in terms of frequency: $2a = 250\,\mu m$, $n_1 = 4$ *and* $n_2 = 3.5$

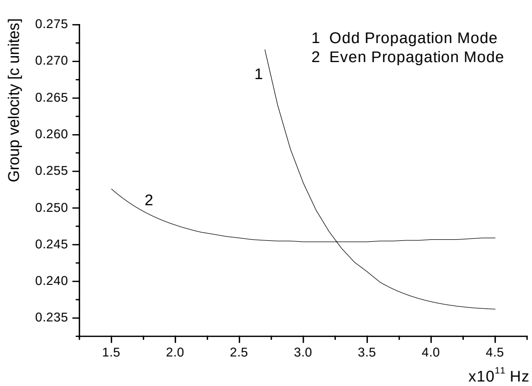

Fig. 4. Group velocity in terms of frequency: $2a = 250\,\mu m$, $n_1 = 4$ *and* $n_2 = 3.5$.

As the last two figures indicate, the penetration depth and the group velocity decrease drastically with the frequency in the investigated domain for the odd propagation modes, in contrast to the even ones.

4. Optical coupling

Last but not least let us consider an optical coupling device as schematized in Fig. 5 with its associated potential in Fig. 7. The corresponding wave functions within the five zones of the potential are tabulated below.

Zone	Wave function
I	$\Psi_I = Ae^{\chi x}$
II	$\Psi_{II} = Be^{ikx} + Ce^{-ikx}$
III	$\Psi_{III} = De^{-\chi x} + Ee^{\chi x}$
IV	$\Psi_{IV} = Fe^{ikx} + Ge^{-ikx}$
V	$\Psi_V = He^{-\chi x}$

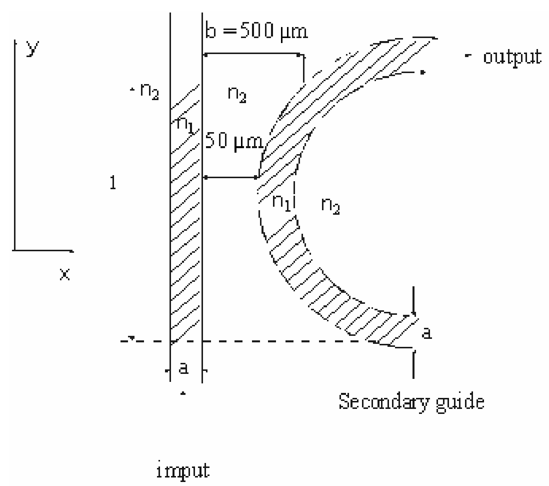

Fig. 5. Optical coupler: $b_1 = 500\ \mu m$, $b_2 = 50\,\mu m$.

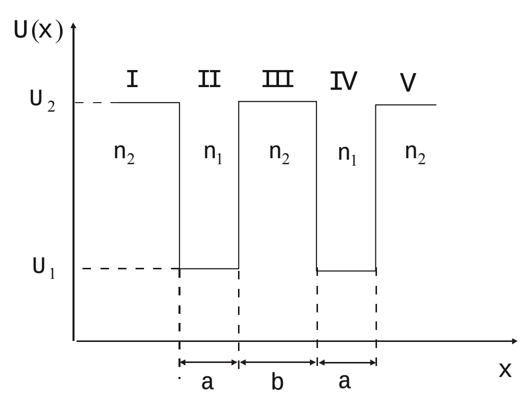

Fig. 6. The potential within the optical coupler cross section.

The continuity requirements for these five wave functions at the four discontinuity locations of the potential allowed us to compute the corresponding photon number or electromagnetic energy density distribution in the transverse section, as given by the square of the wave function modulus, $|\psi|^2$. In the following computation example we will consider a coupler built of fibers having a core breadth of a = 250 μm and a refraction index $n_1 = 4$, and embedded in a dielectric medium of refraction index $n_2 = 3.5$.

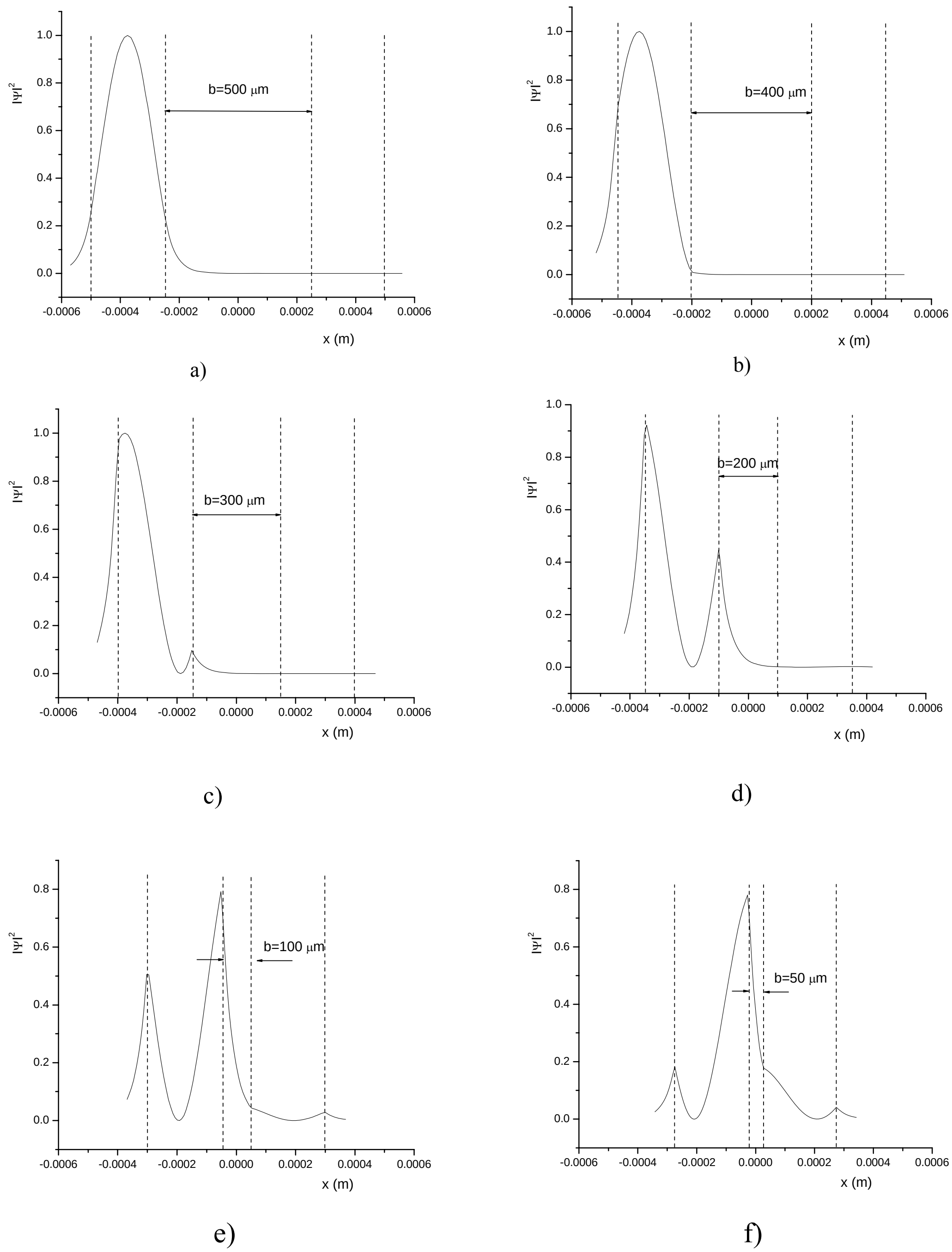

Fig. 7. $|\,\psi\,|^2$ *by optical fiber rapprochement: a) b = 500 μm b) b = 400 μm, c) b = 300 μm, d) b = 200 μm, e) b = 100 μm, f) b = 50 μm.*

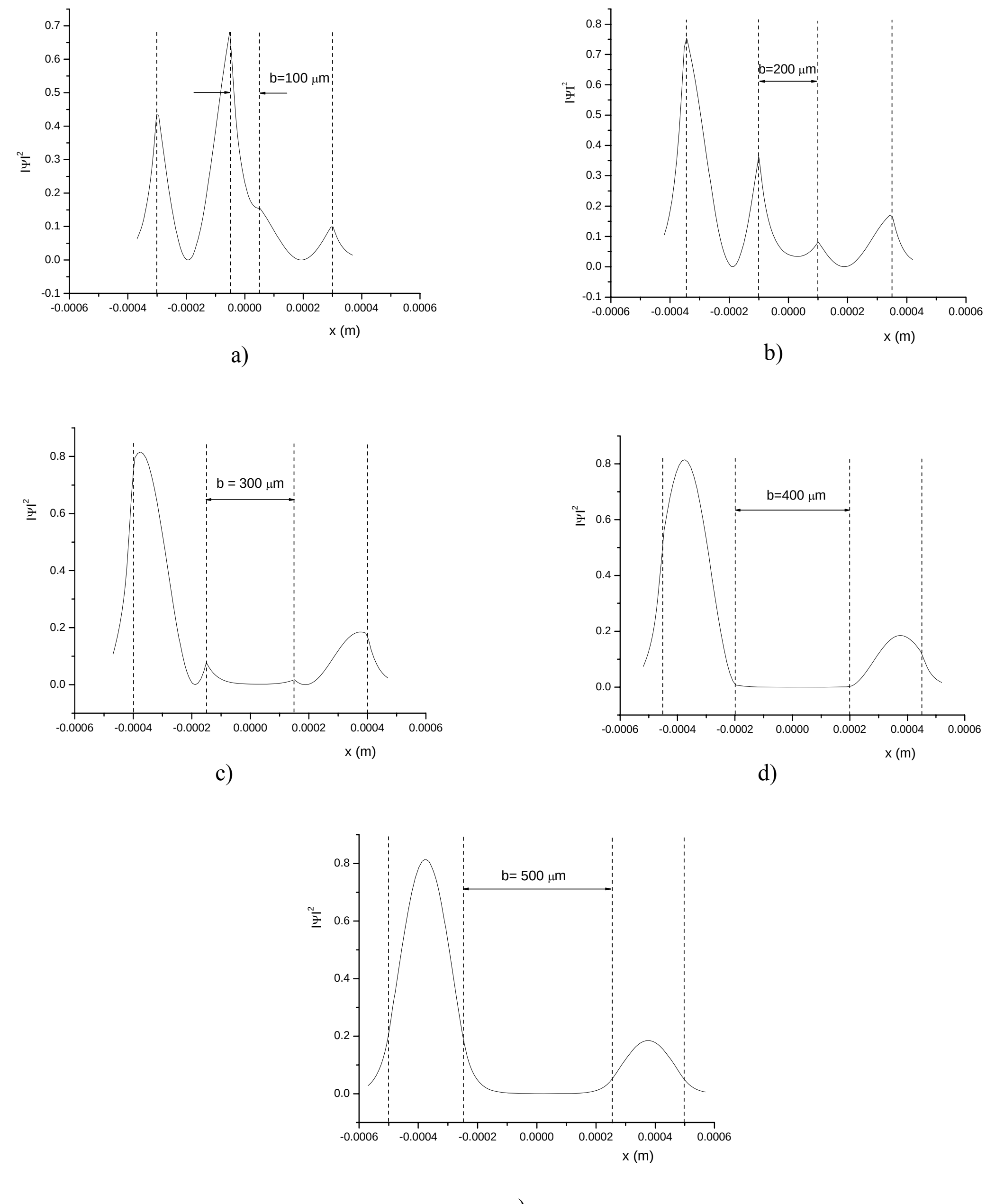

Fig. 8. $|\psi|^2$ *by optical fiber moving off: a) b = 100 μm, b) b = 200 μm, c) b = 300 μm, d) b = 400 μm, e) b = 500 μm.*

This computation has been performed for two distinct cases of primary to secondary optical fiber energy transfer, namely, for rapprochement first, Fig. 7, and then by moving them off, Fig. 8. Generally, at distances greater than 500 μm between the two fibers, the energy transfer between them becomes negligibly small. The minimal distance between the fibers has been limited to 50 μm. Also the coupling length as measured along the Oy axis has been choosen smaller than the wavelength within the guide. The last requirement was necessary in order to ensure a transient mode in the secondary fibre, thus avoiding the installment of a stationary wave within the fiber cross section.

Let us comment in more detail the energy transfer occurring in the considered optical fiber coupler as it is suggested by the $|\psi|^2$ distribution. Thus, in the case illustrated in Fig. 7-f, the secondary wave guide moves towards the primary one from 500 μm up to 50 μm. It may be noticed that at first the maximum of the energy centered in the primary fiber is repelled by the secondary fiber (toward left in Fig. 7 b-c) as if the system would manifest an inertial behavior. However, by continuing the rapprochement (Fig. 7 d-f), it may be noticed the continuous increase of the transferred energy within the secondary fiber, accompanied by a displacement of the energy maximum in the primary fiber in the reverse direction, towards the secondary fiber. As far as the case illustrated in Fig. 8 e is concerned, when the optical fibers are moved off from each other from 50 µm up to 500 µm, it may be observed that the energy maxima in both fibers manifest a reciprocal repulsion (Fig. 8 c-d), until the fibers eventually become decoupled and the corresponding energy maximum occupies its axial position in each fiber.

5. Conclusions

The present paper represent an application of the calculus proposed in our preceding article [1]. It is demonstrated how the information on the electromagnetic energy distribution in the fibers of an optical coupler can be obtained from the sole photon concept, without any appeal to Maxwell equations or from any explicit use of the electric and magnetic field concepts. The computational simplicity is due to the reduction of the number of variables from six in the case of Maxwell equations (three for each of the two fields) to a total of only three, namely the amplitudes of the wave function for three directions. As a rule, in the practice of optical circuits and optoelectronic devices the major interest resides in the knowledge of the energy distribution. We believe that the present paper meets all the needs on this matter.

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*Corresponding author: Rudolf Nistor nistor@physics.pub.ro